TRINITY COLLEGE

# Mathematics Methods Unit 3,4
# Test 2 2019

## Section 1 Calculator Free
**Fundamental Theorem, The Exponential Function & Trigonometry**

**STUDENT'S NAME** ____________________________________________

**DATE:** Thursday 4 April **TIME:** 30 minutes **MARKS:** 32

**INSTRUCTIONS:**
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Given $g'(x) = e^{2x}\cos(2x)$, determine a simplified value for the rate of change of $g'(x)$ when $x = \pi$

$g''(x) = 2e^{2x}\cos 2x - 2e^{2x}\sin 2x$

$g''(\pi) = 2e^{2\pi}\cos 2\pi - 2e^{2\pi}\sin 2\pi$

$= 2e^{2\pi}$

2. (9 marks)

(a) $\frac{d}{dx}\left(\frac{e^{-x}}{\sin x}\right)$ $= \dfrac{-e^{-x}\sin x - e^{-x}\cos x}{\sin^2 x}$ [2]

(b) $\int_0^2 x e^{4-x^2}\,dx$ $= -\frac{1}{2}\int_0^2 -2x\, e^{4-x^2}\,dx$ [3]

$= -\frac{1}{2}\left[e^{4-x^2}\right]_0^2$

$= -\frac{1}{2}\left(e^0 - e^4\right)$

$= -\frac{1}{2}(1 - e^4$

(c) $\frac{d}{dx}\int_x^0 \sin(t) + e^t\ \mathrm{dt}$ $= -\frac{d}{dx}\int_0^x \sin t + e^t\,dt$ [2]

$= -\sin x - e^x$

(d) $\int 4\cos(3x)\ dx$ $= \frac{4}{3}\cos 3x + c$ [2]

3. (4 marks)

Given $\int_1^6 f(x)\, dx = 10$, determine

(a) $\int_6^1 e^2 f(x)\, dx$ [2]

$$= -e^2 \int_1^6 f(x)\, dx$$

$$= -10e^2$$

(b) $\int_1^3 (f(x) - 2)\, dx - \int_6^3 f(x)\, dx$ [2]

$$= \int_1^3 f(x)\, dx - \int_1^3 2\, dx + \int_3^6 f(x)\, dx$$

$$= \int_1^6 f(x)\, dx - \int_1^3 2\, dx$$

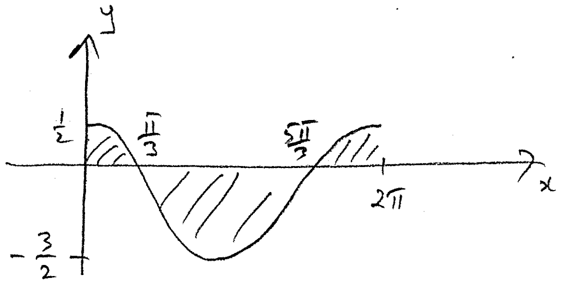
$$= 10 - \left[2x\right]_1^3$$

$$= 10 - (6 - 2)$$

$$= 6$$

4. (5 marks)

Determine the exact area enclosed by $y = \cos x - \frac{1}{2}$ and the x-axis from 0 to $2\pi$




$$\cos x - \frac{1}{2} = 0$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\text{AREA} = 2\int_0^{\frac{\pi}{3}} \cos x - \frac{1}{2}\, dx + \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} \cos x - \frac{1}{2}\, dx$$

$$= 2\left[\sin x - \frac{x}{2}\right]_0^{\frac{\pi}{3}} + \left[\sin x - \frac{x}{2}\right]_{\frac{\pi}{3}}^{\frac{5\pi}{3}}$$

$$= 2\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right) + \left(-\frac{\sqrt{3}}{2} - \frac{5\pi}{6} - \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)\right)$$

$$= 2\sqrt{3} + \frac{\pi}{3}$$

5. (12 marks)

Consider the function: $f(x) = xe^x$.

(a) Determine the co-ordinates of any axis intercepts. [1]

$(0, 0)$

(b) Show that the function has only one stationary point. [3]

$f'(x) = e^x + xe^x$

$e^x + xe^x = 0$

$e^x(1+x) = 0$

$e^x = 0$ NO SOLN $\qquad 1+x=0$, $x=-1$ $\qquad \therefore$ ONLY ONE

(c) Determine the co-ordinates and nature of the stationary point. [3]

STATIONARY POINT

$(-1, -e^{-1})$

$f''(x) = 2e^x + xe^x$

$f''(-1) = 2e^{-1} - e^{-1} > 0$

$\therefore$ MIN

(d) Determine the co-ordinates of the point of inflection. [2]

$f''(x) = 2e^x + xe^x$

$2e^x + xe^x = 0$

$e^x(2+x) = 0$

$e^x = 0$ NO SOLN $\qquad 2+x=0$, $x=-2$

$(-2, -2e^{-2})$

(e) Determine $\int xe^x dx$ [3]

$f'(x) = e^x + xe^x$

$\int f'(x)dx = \int e^x dx + \int xe^x dx$

$f(x) = e^x + \int xe^x dx$

$\therefore \int xe^x dx = xe^x - e^x + c$

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# Mathematics Methods Unit 3,4
# Test 1 2019

## Section 2 Calculator Assumed

**Fundamental Theorem, The Exponential Function & Trigonometry**

**STUDENT'S NAME** ______________________________

**DATE:** Thursday 4 April **TIME:** 20 minutes **MARKS:** 22

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (5 marks)

A radioactive material decays such that it has a half-life of 25 years, meaning it takes twenty-five years for the amount of material to be halved.

(a) State a formula for the mass m(t) remaining after t years where the initial amount is 8000 kg. (Give your answer in the form: $M = M_0e^{kt}$ where $M_0$ and $k$ need to be stated to 4 significant figures). ~~[2]~~ 3

$M = 8000e^{kt}$

$4000 = 8000e^{25k}$

$\therefore k = -0.02773$ $\therefore M = 8000e^{-0.02773k}$

(b) Calculate the mass remaining after 30 years. ~~[3]~~ 2

$M = 8000e^{-0.02773(30)}$

$= 3481.8$

7. (7 marks)

A particle moves along a straight line such that its displacement, $x$ metres at time $t$ seconds is given by the equation $x = 3\sin(2t) + 4$. Determine:

(a) An equation for the velocity of the particle at time $t$. [1]

$v = 6\cos 2t$

(b) The distance from the origin the particle comes to a stop for the first time. [2]

$v = 0$

$6\cos 2t = 0$

$t = \frac{\pi}{4}$

$x = 3\sin\frac{\pi}{2} + 4$

$= 7$

(c) The distance travelled in the first three seconds. [2]

$\int_0^3 |6\cos 2t|\,dt = 11.16$

(d) The acceleration when $t = \frac{3\pi}{4}$ seconds. [2]

$a = -12\sin 2t$

$= -12\sin\frac{3\pi}{2}$

$= 12$

8. (4 marks)
Given that $r = \sqrt{t}$ $t = 4x$ and $x = \cos\theta$ ,

Prove that $\frac{dr}{d\theta} = -\frac{\sin\theta}{\sqrt{\cos\theta}}$

$$\frac{dr}{d\theta} = \frac{dr}{dt} \times \frac{dt}{dx} \times \frac{dx}{d\theta}$$

$$= \frac{1}{2}t^{-\frac{1}{2}} \times 4 \times (-\sin\theta)$$

$$= \frac{-2\sin\theta}{\sqrt{t}}$$

$$= -\frac{2\sin\theta}{\sqrt{\cos\theta}}$$

9. (6 marks)

Element J has a quantity of radioactive material present that has the following relationship $\frac{dA}{dt} = -0.03A$ where A is the amount (in grams) of the element remaining after t days. There is 200 g present initially.

(a) Give an expression for the amount of radioactive material remaining at t days. [2]

$$A = 200e^{-0.03t}$$

(b) How much of the original element is remaining after 2 weeks? [2]

$$A = 200e^{-0.03 \times 14}$$

$$= 131.4$$

(c) Determine when 150 g of material has decayed. [2]

$$A = 50$$

$$50 = 200e^{-0.03t}$$

$$t = 46.21$$

DURING DAY 47